This is a title.

Expected utility theory has been subject to considerable criticism. Perhaps one of the most promising alternative decision-making theory is prospect theory. We examine optimal life cycle behavior under prospect theory preferences. The results show that an agent with these preferences may behave very differently than an agent with standard preferences.

Expected utility theory (EUT) is by far the most dominant paradigm for analyzing (economic) decisions under risk. It goes back at least as far as Gabriel Cramer (1728) and Daniel Bernoulli (1738). They were both seeking a solution to the famous St. Petersburg puzzle, posed in 1713 by Daniel's cousin Nicolas Bernoulli. The St. Petersburg game can be described in the following manner:

Peter tosses a coin and continues to do so until it should land "heads" when it comes to the ground. He agrees to give Paul one ducat if he gets "heads" on the very first throw, two ducats if he gets it on the second, four if on the third, eight if on the fourth, and so on, so that with each additional throw the number of ducats he must pay is doubled. Suppose we seek to determine the value of Paul's expectation (from Bernoulli, 1738, p. 31).

The value of Paul's expectation $\mathbb{E}$ is simply the sum of all individual probabilities multiplied by their corresponding payoffs. More formally,

$$\mathbb{E} = \sum_{k=0}^{\infty} \frac{1}{2^{k+1}} \cdot 2^k = \sum_{k=0}^{\infty} \frac{1}{2} = \infty.$$

Although the value of Paul's expectation $\mathbb{E}$ is infinitely great, no fairly reasonable man would be willing to pay a large amount of ducats to participate in the St. Petersburg game.

Daniel Bernoulli resolved the St. Petersburg puzzle by arguing that the marginal utility of money is not constant, but diminishes. He said: "There is no doubt that a gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount." Specifically, Bernoulli suggested to measure the utility of money on a logarithmic scale rather than a linear scale[1]. Indeed, under logarithmic utility, Paul's expected utility $\mathbb{U}$ is finite:

$$\mathbb{U} = \sum_{k=0}^{\infty} \frac{1}{2^{k+1}} \cdot \log 2^k = \log 2 < \infty.$$

Although Bernoulli resolved the St. Petersburg puzzle, EUT did not receive much attention in the economic literature until the second half of the twentieth century.

Interest in EUT was revived when John von Neumann and Oskar Morgenstern (1944, 1947) showed that EUT can be derived from four axioms of rational choice. These axioms are stated in terms of lotteries, where a lottery L can be represented by a probability distribution $L = (p_1, ..., p_n)$ over a *fixed* set of outcomes $X = (x_1, ..., x_n)$. Here, $p_i \in [0, 1]$ denotes the probability associated with outcome x_i and $\sum_i p_i = 1$. The axioms of rational choice are as follows[2]:

- **Completeness.** For all lotteries L, M: either $L \succeq M$ or $M \succeq L$ or $L \sim M$.
- **Transitivity.** For all lotteries L, M, N: if $L \succeq M$ and $M \succeq N$, then $L \succeq N$.
- **Continuity.** If $L \succeq M \succeq N$, then $\exists\, p \in [0, 1] : pL + (1-p)N \sim M$.
- **Independence.** If $L \succeq M$, then for any N and $p \in [0, 1] : pL + (1-p)N \succeq pM + (1-p)N$.

[1] Daniel Bernoulli does not completely resolve the St. Petersburg puzzle. As long as the utility function is unbounded, the puzzle can be regenerated by redefining the player's payoffs.

[2] The symbol $\succeq$ has the interpretation "at least as good as" and the symbol $\sim$ has the interpretation "equally as good as".

If all four axioms are satisfied, agents' preferences can be represented by:

$$\mathbb{U}(L) = \sum_i p_i \cdot u(x_i),$$

where L is any prospect and $u(\cdot)$ is a utility function. The *expected utility hypothesis* asserts that a person will (weakly) prefer the lottery L over the lottery M if and only if $U(L) \geq U(M)$.

EUT has been subject to considerable criticism, especially relating to the independence axiom. One of the most famous challenges to EUT was presented by the French economist Maurice Allais (1953). He posed a hypothetical pair of decision problems. Consider the following two gambles:

gamble 1A		gamble 2A	
payoff	probability	payoff	probability
\$1M	0.89	\$1M	0.89
\$1M	0.11	\$0	0.01
		\$5M	0.1

If you had to choose, which one would you opt for? Now imagine yourself choosing between the following two gambles:

gamble 1B		gamble 2B	
payoff	probability	payoff	probability
\$0	0.89	\$0	0.89
\$1M	0.11	\$0	0.01
		\$5M	0.1

Which one would you prefer? Several studies have found that most people would choose gamble 1A and gamble 2B. Closer inspection reveals that the second set of gambles is obtained from the first by removing a "common consequence", an 89% chance of winning \$1M. According to the independence axiom of EUT, this operation should have no effect on the relative desirability of one gamble over the other.

A large number of *non-expected utility theories*[3] have been developed to overcome the shortcomings of EUT. Perhaps the most promising non-expected utility theory is cumulative prospect theory (Kahneman and Tversky, 1992). This theory incorporates the following *behavioral* aspects:

1. the carriers of value are gains and losses defined relative to a reference point (i.e. status quo);
2. losses loom larger than equivalent gains, a property called loss aversion;
3. agents are risk-averse in the region of gains and risk-prone in the region of losses;
4. objective probabilities are replaced by subjective decision weights.

Under cumulative prospect theory, agents' preferences can be described in the following way. Let θ be a reference point and π_i a subjective decision weight. For gains (losses), π_i is the difference between the subjective probability of the event "the outcome is at least as good (bad) as x_i" and the subjective probability of the event "the outcome is strictly better (worse) than x_i". Agents' preferences are now given by:

$$\mathbb{U}(L) = \sum_i \pi_i \cdot v(x_i - \theta),$$

where L is any prospect and $v(\cdot)$ is the two-piece power utility function:

$$v(x_i - \theta) = \begin{cases} -\kappa \cdot (\theta - x_i)^{\gamma_1} & \text{if } x_i < \theta \\ (x_i - \theta)^{\gamma_2} & \text{if } x_i \geq \theta \end{cases}. \qquad (1)$$

Here, $\gamma_1, \gamma_2 \in (0, 1]$ are preference parameters and $\kappa > 1$ is the index of loss aversion.

In the sequel, we are interested in optimal consumption and portfolio choice under a general class of reference-dependent preferences. Following Kőszegi and Rabin (2006), we assume that the agent's instantaneous utility function at time $t \in [0, T]$ is given by:

$$w(c_t; \bar{c}_t) = \alpha \cdot u(c_t) + (1 - \alpha) \cdot v(u(c_t) - u(\bar{c}_t)), \qquad (2)$$

where $\alpha \in [0, 1]$ is a weighting parameter, $v(\cdot)$ is the two-piece power utility function (1) and $\bar{c}_t$ is the reference level of consumption. The first term on the right-hand side of (2) represents "consumption utility", that is, utility derived from the absolute level of consumption c_t. In most applications, $u(c_t)$ is assumed to be continuously differentiable, strictly increasing and globally concave. The second term on the right-hand side of (2) captures gain-loss utility, that is, utility derived from the deviation of $u(c_t)$ from the reference level of utility $u(\bar{c}_t)$.

[3] Examples include generalized expected utility theory, weighted utility theory, quadratic utility theory and rank-dependent expected utility theory.

For the sake of simplicity, we make the assumption that the reference level of consumption is exogenously determined: the agent enters the labor market with a given prior belief about his future consumption levels.

The agent's lifetime utility function is given by[4]:

$$\mathbb{V}(c;\bar{c}) = \int_0^T e^{-\delta t} \int_0^\infty w(c_t;\bar{c}_t)\mathrm{d}T(F_{c_t}(c_t))\mathrm{d}t, \quad (3)$$

where $\delta \geq 0$ is the rate of time preference, $w(\cdot;\cdot)$ is given by (2), $T : [0,1] \mapsto [0,1]$ is a probability weighting function and $F_{c_t}(\cdot)$ is the cumulative distribution function of c_t.

The agent wishes to maximize his lifetime utility function (**??**) over admissible consumption and portfolio strategies subject to his dynamic budget constraint[5]. In a complete market setting, certain special cases of the optimization problem can be solved by using the martingale method of Cox and Huang (1989).

We illustrate the optimal consumption profile for a special case where:

$$u(c_t) = \frac{1}{1-\eta}c_t^{1-\eta}; \qquad \gamma_1 = \gamma_2 = 1; \qquad T(p) = p.$$

Figure 1 displays the optimal consumption profile[6] as a function of the state price density. For the sake of comparison, we also illustrate the optimal consumption profile for an agent with only "consumption utility" (red dashed line). As can be seen in Figure 1, the optimal consumption profile falls into three regions. In good states of the world (i.e. low state prices), the agent gives up some upward potential in order to protect himself against low consumption levels. In intermediate states of the world (i.e. moderate state prices), the optimal consumption profile is constant at the reference level. In bad states of the world (i.e. high state prices), the agent's wealth is insufficient to finance consumption at the reference level.

The above discussion reveals that an agent with alternative preferences may behave very differently than an agent with standard preferences. Further research is needed to completely understand the effects of e.g. subjective probability weighting, loss aversion and reference-point formation on optimal life cycle behavior.

[4]We abstract from longevity risk and bequest motives.

[5]We assume that the agent invests his wealth in a financial market consisting of one risky stock and one risk-free asset.

[6]The optimal portfolio strategy is the one that replicates the optimal consumption strategy.

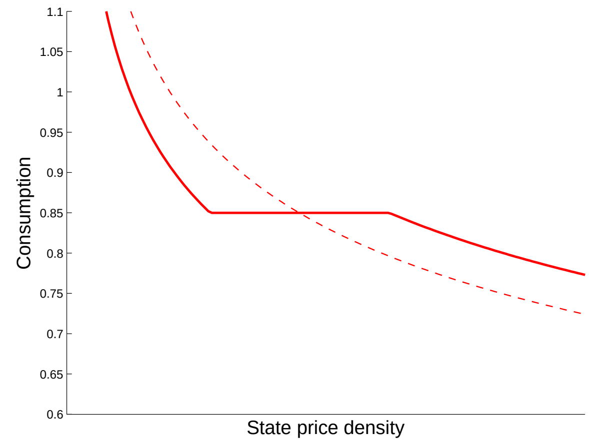

Figure 1: Optimal consumption profile.
This figure displays the optimal consumption profile for an agent with reference-dependent preferences (red solid line) and an agent with only "consumption utility" (red dashed line). Consumption levels are expressed in terms of labor income. The underlying preference parameters are as follows: $(\bar{c}_t, \delta, \alpha, \eta, \kappa) = (0.85, 0.04, 0.5, 5, 4.5)$.

References

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