Over-The-Counter Trades in Longevity Risk

There is an increasing need for hedging non-marketed risk. Markets serve as mechanism to reallocate risk among firms. For longevity risk, however, there does not exist a well-functioning market. As an alternative, firms could benefit from risk sharing by trading Over-The-Counter. I examine the question how to design such a contract.

Pension funds and insurance companies hold risky portfolios. These firms face a variety of risks such as inflation, interest and longevity risk. Generally, firms decide to hedge their risk in the market. For instance, for inflation and interest, there exists a well-functioning market so that prices exist. In this way, firms optimally compose their portfolio of financial assets in such a way that the exposure to these risks is acceptable. In this essay, I focus on longevity risk. For longevity risk, there exists no well-functioning market. Therefore, market prices hardly exist and classical asset pricing theory cannot be applied. This essay is based on Boonen et al. (2012) and Boonen (2012).

Longevity risk is the risk that people in the population live longer or shorter than expected. I focus on the trend of mortality fluctuations over time. If people live longer than expected, a pension fund has to pay the promised pension benefits for a longer period of time. The liabilities, which are given by the value of the current pension promises, are therefore increasing if people live longer than expected. Longevity risk affects the liabilities of life insurers in a different manner. Life insurance is often obligatory if individuals buy a mortgage. Moreover, mortgages are often repaid at the retirement age (via tax incentives) and, so, life insurance contracts terminate at this age as well. Therefore, life insurers mainly face the risk of early death of their policy holders. Hence, a life insurer faces the risk that people live shorter than expected.

Exposure to longevity risk can be rather substantial for pension funds, as shown by, e.g., Hári et al. (2008). Lack of consensus regarding accurate pricing hampers trade. As a consequence, longevity-linked contracts are mainly traded Over-The-Counter (OTC). This implies that two firms trade directly without any supervision of an exchange. In the recent history, there only have been a few Over-The-Counter trades in this risk. As pointed out by Dowd et al. (2006), the market for Over-The-Counter longevity contracts is expected to grow fast. Firms can naturally hedge longevity risk by combining pension policies and life insurance policies, but there is little capacity in the market to do so. The conflicting exposure to longevity risk for pension funds and life insurers creates incentives for both firms to trade. I analyze the opportunities for a pension fund and a life insurer to benefit from bilateral risk redistribution via Over-The-Counter trade. Specifically, Over-The-Counter contracts describe a specific risk redistribution, i.e., in every possible scenario it is described a priori how much a firm pays or receives from the other firm. This implies that prices are determined implicitly.

I consider the case where a pension fund and a life insurer redistribute risk in order to reduce the volatility of their Net Asset Value (NAV) at a prespecified future date T. The Net Asset Value is defined as the difference between the value of the assets and the value of the liabilities, i.e.,

$$X_i(T) = A_i(T) - L_i(T), \qquad (1)$$

where firm i is either a pension fund or a life insurer, $A_i(T)$ denotes the (market) value of the assets at time T and $L_i(T)$ denotes the date-T value of the liabilities.

In order to focus on longevity risk, I assume a deterministic return on assets r. The asset value on date T then follows from

$$A_i(t) = (1+r) \cdot A_i(t-1) - \widetilde{L}_{i,t}, \qquad (2)$$

for all $t = 1, \ldots, T$, where $\widetilde{L}_{i,t}$ denotes the (stochastic) liability payment of firm i at date

t. The exact structure of the risky payment $\widetilde{L}_{i,t}$ depends on the type of firm (pension fund or life insurance) and the policies themselves. Combining (1) and (2) yields

$$X_i(T) = [A_i(0) - CL_i(T)] \cdot (1+r)^T, \tag{3}$$

where

$$CL_i(T) = \sum_{\tau=1}^{T} \frac{\widetilde{L}_{i,\tau}}{(1+r)^\tau} + \frac{L_i(T)}{(1+r)^T}. \tag{4}$$

Thus, firm i's Net Asset Value equals the initial asset value $A_i(0)$ increased by the return on assets, and reduced by the random variable $CL_i(T) \cdot (1+r)^T$, where $CL_i(T)$ represents the sum of the present value of liability payments up to year T, and the present value of the date-T liability value of all payments beyond date T.

It now remains to specify how the date-T liability value $L_i(T)$ is determined. Because there is (not yet) a liquid market for longevity-linked products, pension funds and life insurers cannot valuate their liabilities using market prices. Instead, I consider the case where the pension fund and the life insurer valuate their liabilities at the *best estimate value*. The best estimate value of the liabilities is defined as the discounted expected value of all future claims, i.e.,

$$L_i(T) = E\left[\sum_{\tau \geq 1} \frac{\widetilde{L}_{i,T+\tau}}{(1+r)^\tau} \middle| \mathcal{F}_T\right], \tag{5}$$

for all $i \in \{1,2\}$ and $T \geq 0$, where $\mathcal{F}_T$ denotes the information available on date T (i.e., information regarding mortality rates). On date zero, the risk $X_i(T)$ is uncertain due to uncertainty in the liability payments $\widetilde{L}_{i,\tau}$, for $\tau = 1, \ldots, T$, as well as due to uncertainty in the value of the remaining liabilities on date T, $L_i(T)$.

Next, I discuss the preferences of the firms. A firm aims at a portfolio after redistribution with a high expected value and low risk. If both criterions conflict, the firm optimizes an appropriate trade-off. Risk is generally measured using a risk measure. I let this risk measure be Expected Shortfall. This risk measure focusses on worst-case possible realizations and is given by:

$$\rho_i(X) = E[X | X \leq q_{\alpha_i}(X)], \tag{6}$$

where $q_{\alpha_i}(X)$ is the α_i-quantile of risk X. The introduction of the Basel II regulation and the Swiss Solvency Test (SST) has increased the use of Expected Shortfall to evaluate financial or insurance risk. Expected Shortfall attempts to reflect business practices as it has been gaining practitioner interest. This leads to the following preference function for firm i:

$$V_i(X) = E[X] - \beta_i \cdot \rho_i(X), \tag{7}$$

where $\beta_i \geq 0$ represents the relative weight to the risk instead of the expected value. Firm i prefers risk X over risk Y if and only if $V_i(X) \geq V_i(Y)$. I allow that firms use heterogeneous α_i and β_i.

I consider optimal bilateral risk sharing of longevity risk between a pension fund (denoted as Firm 1) and a life insurer (denoted as Firm 2). The firms hold risk $X_1(T)$ and $X_2(T)$ respectively, which is specified in (1). Via an Over-The-Counter contract, both firms want to arrive at a risk redistribution $(X_1^{\text{post}}(T), X_2^{\text{post}}(T))$ such that they both benefit weakly, i.e.,

$$V_i(X_i^{\text{post}}(T)) \geq V_i(X_i(T)), \text{ for all } i = 1, 2, \tag{8}$$

and all risk is redistributed, i.e.,

$$X_1^{\text{post}}(T) + X_2^{\text{post}}(T) = X_1(T) + X_2(T). \tag{9}$$

The risk redistribution takes place at date T; the date at which the realization of $X_i(T)$ is known.

A very intuitive criterion for a risk redistribution is Pareto optimality. Pareto optimal risk redistributions are defined as the ones such that there does not exist another risk redistribution that is weakly better for all firms and strictly better for at least one firm. Generally, there are still infinitely many Pareto optimal risk redistributions satisfying (8). It holds that $(X_1^{\text{post}}(T), X_2^{\text{post}}(T))$ is Pareto optimal if and only if $(X_1^{\text{post}}(T) + c, X_2^{\text{post}}(T) - c)$ with $c \in \mathbb{R}$ is Pareto optimal. Therefore, I first determine a Pareto optimal risk redistribution $(X_1^{\text{post}}(T), X_2^{\text{post}}(T))$ and, thereafter, a corresponding "side-payment" c.

In two special cases, I obtain the following Pareto optimal risk redistributions:

1. if $\alpha_1 = \alpha_2$, a Pareto optimal solution is to shift all risk to the firm with the smallest β_i;
2. if $\beta_1 = \beta_2$, a Pareto optimal solution is to shift all risk to the firm with the largest α_i.

Particularly, 1. implies that if the risk measure is set by a (common) regulator, it is likely that there exists a firm that will be overexposed to longevity risk. Note that I provide a (non-unique) Pareto optimal risk redistribution. If there are variable parameters α_i and β_i, there always exists a Pareto optimal risk redistribution that is given by a stop-loss contract on the aggregate risk with a given threshold. So, there exists a firm $i \in \{1,2\}$ that buys deductible insurance, i.e.,

$$X_i^{\text{post}}(T) = \max\{X_1(T) + X_2(T) - d, 0\}. \tag{10}$$

for some threshold d. The other firm (not i) buys the rest of the aggregate risk, i.e.,

$$X_{-i}^{\text{post}}(T) = \min\{X_1(T) + X_2(T), d\}, \tag{11}$$

Particularly for insurance risk, stop-loss contracts are often observed in Over-The-Counter trades.

A Pareto optimal risk redistribution is provided in (10) and (11). This risk redistribution does not need to be beneficial for both firms. Therefore, one firm might require a side-payment on top of this risk redistribution, which is a risk-free payment. There is a non-empty interval of possible prices satisfying also (8). A question left to determine is the size of this side-payment in this interval that is perceived as "fair" by both firms. I propose to use a cooperative game-theoretic characterization of this side-payment.

Next, I work out an example. I simulate the liabilities of both the pension fund and the life insurer using a realistic composition of the policy holders and a realistic underlying longevity distribution. The return on assets equals $r = 3\%$. I here set $T = 1$, and show that the benefits from redistribution are significant even when the horizon is relatively short. Because firms are reluctant to engage in contracts with a long horizon, a one-year horizon is appealing. Moreover, in the future, firms are willing to hedge their longevity risk in the new policies again. It is sufficient to consider only the liabilities of both firms. The objective function of the pension fund is given by (7) with $\alpha_1 = 50\%$ and $\beta_1 = 3$. The pension fund sets an ambition level, which is the expected value. Moreover, the risk measure is the expected value of all outcomes given that it is below the ambition level. The objective function of the life insurer is given by (7) with $\alpha_2 = 1\%$ and $\beta_2 = 1$. This exactly meets the Swiss Solvency Test (SST) regulation for insurance companies. I display the prior and posterior date-1 value of the current liabilities in Figure 1. This figure displays that the risk reduction is significant. The life insurer has relatively volatile liabilities before redistribution, which is mitigated thereafter. The tail-risk of the aggregate liabilities is borne by the pension fund.

Finally, I calculate the welfare gains from this risk redistribution. This is the relative price that firms would be willing to pay for the risk redistribution compared with the price of the prior liabilities. This is given by

$$\frac{V_i(X_i^{\text{post}}(1)) - V_i(X_i(1))}{V_i(X_i(1))} = \begin{cases} 0.6\% & \text{if } i = 1, \\ 6.2\% & \text{if } i = 2. \end{cases} \qquad (12)$$

The pension fund (resp. life insurer) is willing to pay 0.6% (resp. 6.2%) of its current liabilities for the risk redistribution. As there are no prices involved in the risk redistribution, these percentages indicate the welfare gains. The life insurer gets larger welfare gains due to his relative volatile prior liabilities and due that the aggregate risk $X_1(1) + X_2(1)$ is small in states where the prior risk $X_2(1)$ is large and vice versa. The percentages in (12) indicate that the potential for OTC trades

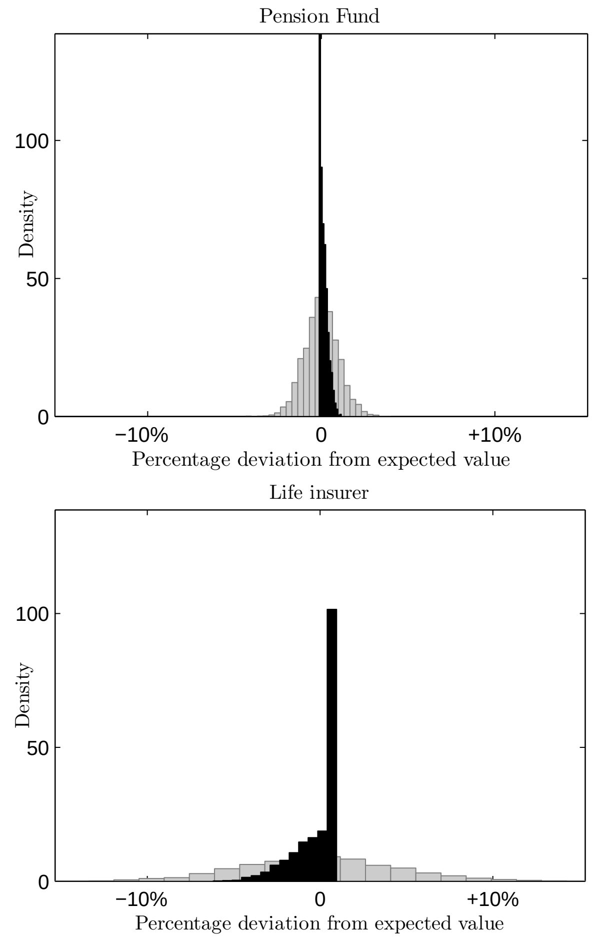

Figure 1: Prior (gray histogram) and posterior (black histogram) date-1 value of the current liabilities of the pension fund (upper figure) and the life insurer (lower figure). Note that liabilities are interpreted as losses instead of gains.

is substantial and, therefore, I expect the market for longevity risk to grow quickly.

References

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